Wireless Network Pricing
Chapter 3: Economics Basics

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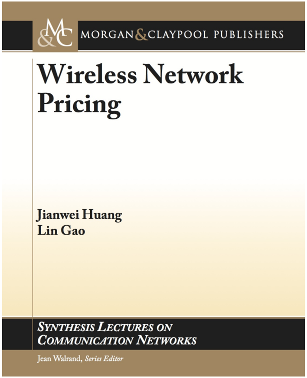
The Book



- E-Book freely downloadable from NCEL website: `http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing`
- Physical book available for purchase from Morgan & Claypool (`http://goo.gl/JFGlai`) and Amazon (`http://goo.gl/JQKaEq`)

Chapter 3: Economics Basics

What is Economics?

Definition (Economics)

Economics is the study of how individuals and groups make decisions with limited resources as to best satisfy their wants, needs, and desires.

Firm and Consumer

- Convention terminologies: "firm" and "consumer"
  - Examples of firm: wireless service provider
  - Examples of consumer: mobile user

Definition (Firm)

A firm is an organization involved in the production and trade of goods, services, or both to consumers.

Definition (Consumer)

A consumer is a person or group of people, such as a household, who are the users of products or services.

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Definition (Consumer)

A consumer is a person or group of people, such as a household, who are the users of products or services.

- Question: Consider the case where an MNO (PCCW) sells spectrum to a MVNO (China Unicom HK), who in turn serves the mobile users. Who is the firm and who is the consumer?

Examples: Economics

Ballard Farmers' Market (source: Internet)

Examples: Economics

Sao Paulo Stock Exchange (source: Internet)

Examples: Economics

Christie's Auction (source: Internet)

Section 3.1: Supply and Demand

Supply and Demand

- Supply and Demand in a market are both functions of market prices.
- Demand (of consumers) often decreases with prices, as consumers have less incentives to purchase under higher prices.
- Supply (of firms) often increases with prices, as firms have more incentives to produce under higher prices.

Market Demand Function

Example

- A consumer subscribes to a monthly wireless cellular data plan.
  - Consumer's demand is 50 Gigabytes, if the price is \$1/GB;
  - Consumer's demand is 1.5 Gigabytes, if the price is \$20/GB.

Price (\$/GB)	Monthly Wireless Data Demand
1	50 GB
2	22 GB
10	4 GB
20	1.5 GB

Market Demand Function

- Market Demand Function: The relationship between the aggregate demand (of all consumers) and the market price.

Definition (Market Demand Function)

The market demand function $D(\cdot)$ characterizes the relationship between the total demand quantity Q_d and the product price P as follows:

$$Q_d = D(P)$$

Market Demand Function

- Illustration of Market Demand Function

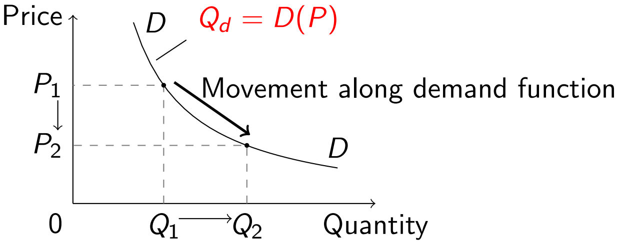


Figure: The market demand function $Q_d = D(P)$. When the price decreases from P_1 to P_2, the demand increases from Q_1 to Q_2.

Market Demand Function

- Illustration of Market Demand Function

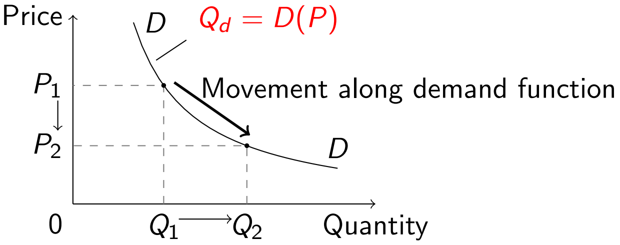


Figure: The market demand function $Q_d = D(P)$. When the price decreases from P_1 to P_2, the demand increases from Q_1 to Q_2.

- Question: How to draw the market demand function of the previous monthly data plan example?

Market Demand Function

- Market demand function itself may shift due to
  - the change of consumers' income;
  - the price change of other products;
  - the change of consumers' tastes;

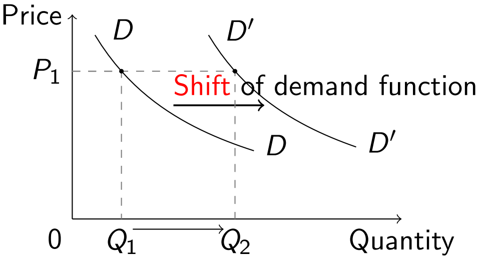


Figure: The shift of market demand function from $Q_d = D(P)$ to $Q_d' = D'(P)$. For example, under the same price P_1, the demand changes from Q_1 to Q_2.

Market Supply Function

- Market Supply Function: The relationship between the aggregate supply (of all firms) and the market price.

Definition (Market Supply Function)

The market supply function $S(\cdot)$ characterizes the relationship between the total supply quantity Q_s and the product price P as follows

$$Q_s = S(P)$$

Market Supply Function

- Illustration of Market Supply Function

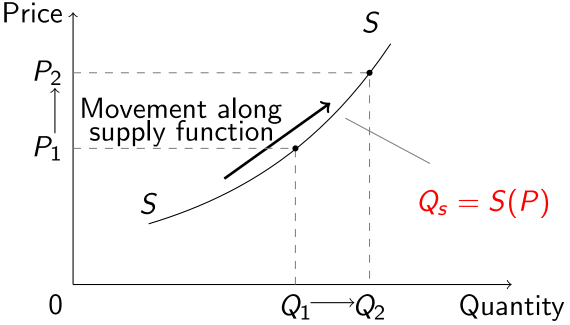


Figure: The market supply function $Q_s = S(P)$. When the price increases from P_1 to P_2, the supply increases from Q_1 to Q_2.

- Market supply function itself may shift when the price of a raw material (used for production) or the production technology changes.

Market Equilibrium

- Market Equilibrium: A market stable state under which the market is unlikely to change.
  - A prediction of how the actual market will look.
- A market (or market price) is unstable, when
  - The aggregate demand $>$ the aggregate supply
    $\Rightarrow$ consumers are willing to pay more to secure the limited supply
    $\Rightarrow$ market price increases
  - The aggregate demand $<$ the aggregate supply
    $\Rightarrow$ firms are willing to charge less to attract the limited demand
    $\Rightarrow$ market price decreases

Market Equilibrium

- Illustration of Market Equilibrium

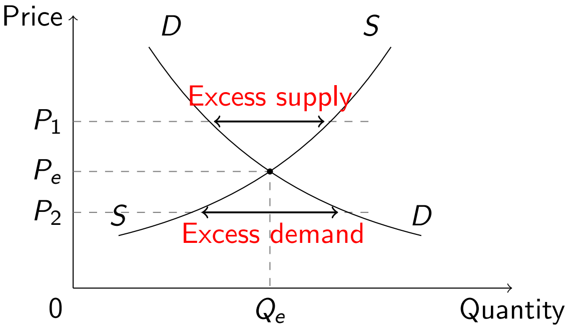


Figure: The market equilibrium price P_e and equilibrium quantity Q_e.

- When either market demand or supply function shifts due to factors other than the price, market equilibrium will change accordingly.

Market Equilibrium

Definition (Market Equilibrium)

At the market equilibrium, the aggregate demand equals the aggregate supply.

- Market equilibrium price P_e and the aggregate demand/supply Q_e:

$$Q_e = D(P_e) = S(P_e)$$

Section 3.2: Consumer Behavior

Consumer Behavior

- Focus on the behavior of a particular consumer
- Understand how to derive the market demand function $Q_d = D(P)$ from the consumer's utility maximization behavior.

Consumer Behavior

- Focus on the behavior of a particular consumer
- Understand how to derive the market demand function $Q_d = D(P)$ from the consumer's utility maximization behavior.
- Basic Concepts
  - Market Basket
  - Consumer Utility
  - Indifference Curve
  - Budget Constraint
  - Consumer Demand Function
  - Price Elasticity

Market Basket

- How would a consumer evaluate the benefit of consuming products?
  - Example: how would a consumer evaluate the total satisfaction level of watching a 60-minute action movie and playing 30 minutes of video games on iPad?

Definition (Market Basket)

A market basket (also known as commodity bundle) specifies the quantity of different products.

Example

Watching a 60-minute movie and playing 30 minutes of game can be represented by the market basket $(60, 30)$.

Consumer Utility

- Consumer Utility Function: Characterize a consumer's satisfaction level of consuming a certain market basket (x, y), i.e.,

$$U = U(x, y)$$

Indifference Curve

- Indifference Curve: Characterizes how a consumer trades off two different baskets of products

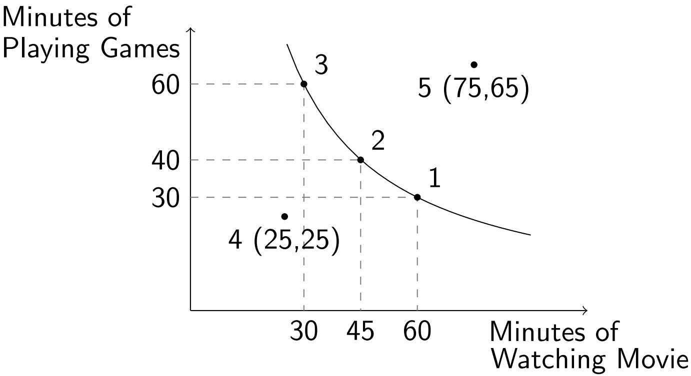
Definition (Indifference Curve)

An indifference curve represents a set of market baskets where the consumer's utilities are the same.

Indifference Curve

Example

- Basket 1 $(60, 30)$, basket 2 $(45, 40)$, and basket 3 $(30, 60)$ are on the same indifference curve (benchmark);
- Basket 5 $(75, 65)$ is on an indifference curve with a higher utility;
- Basket 4 $(25, 25)$ is on an indifference curve with a lower utility.



Budget Constraint

Definition (Budget Constraint)

The budget constraint characterizes which market baskets are affordable to the consumer.

Example

- Watching one minute of movie will cost 1 unit of energy
- Playing one minute of game will cost 2 units of energy
- The constraint of 100 units of energy leads to the budget constraint:

$$x + 2y \leq 100$$

- More generally,

$$P_x x + P_y y \leq I$$

  - Here P_x and P_y are the unit prices, and I is the budget.

Budget Constraint

- Illustration of Budget Constraint

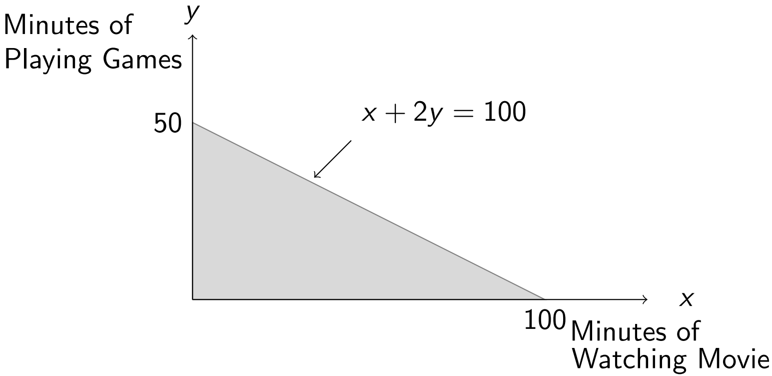


Figure: Illustration of budget constraint $x + 2y \leq 100$.

Consumer Consumption Problem

- How does a consumer decide which market basket to purchase?
- Consumer's objective: maximize his utility subject to the budget constraint.
- Geometrically, the consumer's optimal choice is the highest indifference curve that "touches" the budget constraint.

Consumer Consumption Problem

- Illustration of Consumer's Optimal Choice
  - $U_1 < U_2 < U_3$ are three indifference curves;
  - Budget constraint is $x_c + 2y_c \leq 100$;

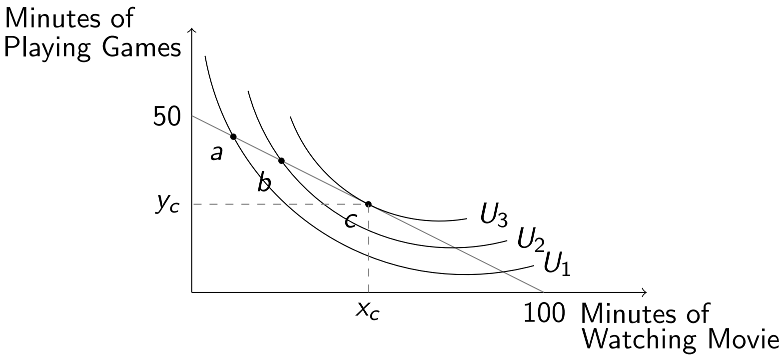


Figure: Consumer's optimal market basket choice is basket c.

Consumer Consumption Problem

- Consumer's Optimal Choice in the previous figure
  - The budget constraint is

$$P_x x + P_y y \leq I$$

  - The budget constraint is the tangent line to the indifference curve U_3 at the market basket c:

$$\left.\frac{\Delta y}{\Delta x}\right|_{U(x,y)=U_3,(x,y)=(x_c,y_c)} = -\frac{P_x}{P_y}$$

  - The LHS (left hand side) is the marginal rate of substitution (MRS)
    - ★ Representing how much the consumer is willing to tradeoff one product with the other product.
    - ★ In general MRS is not a constant on a particular indifference curve.

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    - ★ In general MRS is not a constant on a particular indifference curve.
  - Question: How does the consumer trade off playing games and watching movies?

Consumer Demand Function

- Consumer Demand Function: Characterizes how a consumer's demand of a product changes with the price of that product.
- Market demand function: the summation of all consumers' demand functions in the same market.

Consumer Demand Function

- Example: Assume that there are three games on iPad.
  - The consumer can choose one game to play and watch movie.
  - The energy prices of these three games are 1/min, 2/min, and 4/min, respectively. The energy price of watching movie is 1/min.

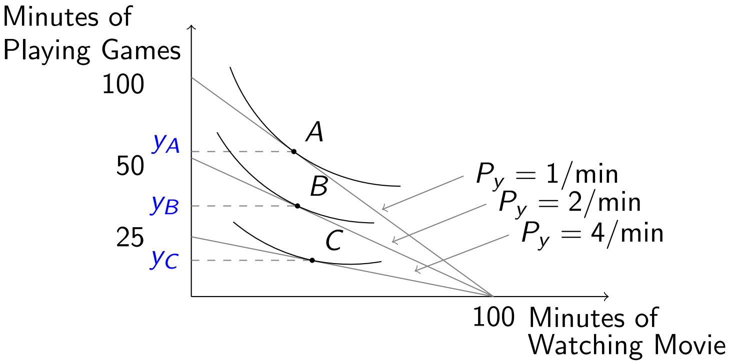

Figure: Consumer's optimal choices: A for 1/min, B for 2/min, C for 4/min.

Consumer Demand Function

- Connecting the consumer's optimal choices under different energy prices will lead to the demand function.

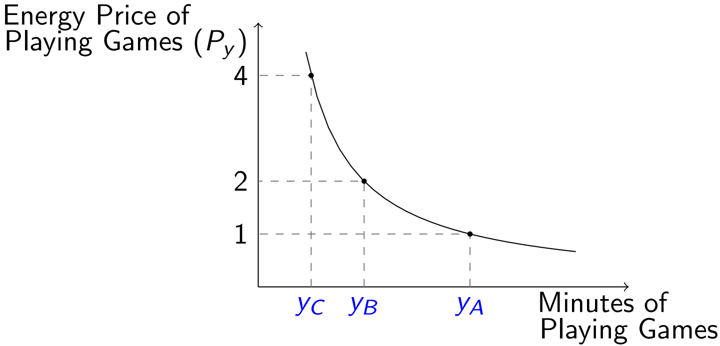

Figure: Consumer's demand function (for playing games) as a function of the energy price.

Price Elasticity

- Price Elasticity: Characterize the sensitivity of demand in term of price, i.e., how fast the demand changes with the price.
- Example: cellular wireless data usage
  - A college student might be very price sensitive, and will dramatically decrease the monthly data usage if the price increases;
  - A CEO might be much less sensitive and not even notice the change of price until several months later.

Price Elasticity

Definition (Price Elasticity)

The price elasticity of demand measures the ratio between the percentage change of demand and the percentage change of price, i.e.,

$$E_d = \frac{\%\text{ change in demand}}{\%\text{ change in price}} = \frac{\Delta Q_d / Q_d}{\Delta P / P}$$

Price Elasticity

- Illustration of Price Elasticity E_d
  - $E_d < 0$ due to the downward slopping of the demand curve.

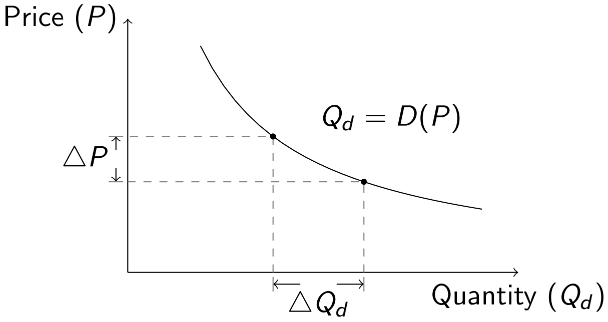

Figure: The change of demand ΔQ_d due to the change of price ΔP.

Price Elasticity

- When the demand function Q_d is differentiable, then

$$E_d = \frac{P}{Q_d} \cdot \frac{\partial Q_d}{\partial P}$$

- Three Demand Types:
  - Elastic demand: the demand changes significantly with the price and $|E_d| > 1$.
  - Inelastic demand: the demand is not sensitive to price and $0 < |E_d| < 1$.
  - Unitary elastic demand: $|E_d| = 1$.

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- Questions: Can you give some examples?

Section 3.3: Firm Behavior

Firm Behavior

- Focus on the behavior of a particular firm
- Understand how to derive the market supply function $Q_s = S(P)$ from the firm's cost minimization behavior
- Basic Concepts
  - Marginal Cost
  - Competitive Firm

Firm Cost

- Total Cost of A firm:
  - Fixed cost: the cost independent of the quantity produced.
  - Variable cost: the cost depending on the production quantity.

Definition (Firm Cost)

The total production cost of a firm includes both the fixed cost F and variable cost $V(q)$, i.e.,

$$C(q) = F + V(q),$$

where q is the production quantity.

Marginal Cost

- Marginal Cost: Characterize how the total cost $C(q)$ changes when the firm changes the production quantity q.

Definition (Marginal Cost)

The marginal cost measures how the total cost changes with the production quantity, i.e.,

$$MC(q) = \frac{\text{change in total production cost}}{\text{change in production quantity}} = \frac{\Delta C(q)}{\Delta q} = \frac{\Delta V(q)}{\Delta q}$$

- The fixed cost F does not affect the computation of marginal cost.
- If the variable cost function $V(q)$ is differentiable, then

$$MC(q) = \frac{\partial C(q)}{\partial q} = \frac{\partial V(q)}{\partial q}$$

Competitive Firm

Definition (Competitive Firm)

A competitive firm is price-taking and acts as if the market price is independent of the quantity produced and sold by the firm.

- The above definition reflects the reality when the firm faces many competitors in the same market.
- Each firm's production decision is unlikely to significantly change the total quantity available in the market, and thus will not significantly affect the market price.
- In Chapter 6, we will talk about the case where the market price changes with the production quantity (e.g., Cournot competition).

Competitive Firm Profit

- Total Profit of a Competitive Firm
  - q: the firm's production quantity;
  - P: the market price independent of the quantity q;
  - F: the firm's fixed cost independent of the quantity q;
  - $V(q)$: the firm's variable cost depending on the quantity q;

Definition (Profit of Competitive Firm)

A competitive firm's total profit is the difference between the total revenue and total cost, i.e.,

$$\pi(q) = P \cdot q - V(q) - F$$

Competitive Firm's Optimal Decision

- A Competitive Firm's Decision Problem: Decide the optimal production quantity q that maximizes its total profit:

$$\pi(q) = P \cdot q - V(q) - F \tag{1}$$

- Set the first order derivative of equation (1) to be zero
$\Rightarrow$ The Firm's Optimal Quantity Choice q^* satisfies:

$$P = \left.\frac{\partial V(q)}{\partial q}\right|_{q=q^*} = MC(q^*)$$

- The Firm's Supply Function is the firm's inverse marginal cost function

$$Q_s = MC^{-1}(P) = S(P)$$

Example

Example

Assume the market price $P = 2$, variable cost $V(q) = q^2$, and fixed cost $F = 0$. What will be the firm's optimal quantity choice?

- Solution:

$$\max_{q \geq 0} \pi(q) = \max_{q \geq 0} (P \cdot q - V(q) - F) = \max_{q \geq 0} \left(2 \cdot q - q^2\right)$$

  - First order condition: $p = MC(q)$, which is $2 = 2q^*$.
  - Hence $q^* = 1$. The total profit is $2 - 1 = 1$.
  - The firm's supply function $Q_s = S(P) = MC^{-1}(P) = P/2$.

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- Questions:
  - What if $V(q) = 3q$?

Section 3.4: Chapter Summary

Key Concepts

- Supply and Demand
- Consumer Behavior Model
- Firm Behavior Model

Extended Reading

http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing