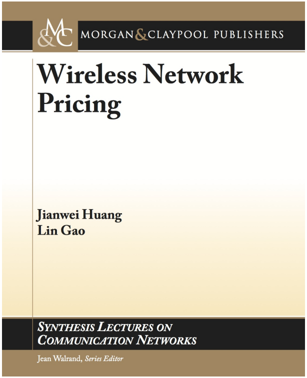
Wireless Network Pricing
Chapter 6: Oligopoly Pricing

Jianwei Huang & Lin Gao

Network Communications and Economics Lab (NCEL)
Information Engineering Department
The Chinese University of Hong Kong

The Book



- E-Book freely downloadable from NCEL website: http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing
- Physical book available for purchase from Morgan & Claypool (http://goo.gl/JFGlai) and Amazon (http://goo.gl/JQKaEq)

Chapter 6: Oligopoly Pricing

Section 6.2
Theory: Oligopoly

Oligopoly

- In this part, we consider three classical strategic form games to formulate the interactions among multiple competitive entities (Oligopoly):
  - The Cournot Model
  - The Bertrand Model
  - The Hotelling Model
- Our purpose in this part is to illustrate
  - (a) Game Formulation: the translation of an informal problem statement into a strategic form representation of a game;
  - (b) Equilibrium Analysis: the analysis of Nash equilibrium when a player can choose his strategy from a continuous set.

The Cournot Model

- The Cournot model describes interactions among firms that *compete on the amount of output they will produce*, which they decide independently of each other simultaneously.

- Key features
  - At least two firms producing homogeneous products;
  - Firms do not cooperate, i.e., there is no collusion;
  - Firms compete by setting production quantities simultaneously;
  - The total output quantity affects the market price;
  - The firms are economically rational and act strategically, seeking to maximize profits given their competitors' decisions.

The Cournot Model

- Example: The Cournot Game
  - Two firms decide their respective output quantities simultaneously;
  - The market price is a decreasing function of the total quantity.
- Game Formulation
  - The set of players is $\mathcal{I} = \{1, 2\}$,
  - The strategy set available to each player $i \in \mathcal{I}$ is the set of all nonnegative real numbers, i.e., $q_i \in [0, \infty)$,
  - The payoff received by each player i is a function of both players' strategies, defined by

$$\Pi_i(q_i, q_{-i}) = q_i \cdot P(q_i + q_{-i}) - c_i \cdot q_i$$

    - ★ The first term denotes the player i's revenue from selling q_i units of products at a market-clearing price $P(q_i + q_{-i})$;
    - ★ The second term denotes the player i's production cost.

The Cournot Model

- Consider a linear cost: $P(q_i + q_{-i}) = a - (q_i + q_{-i})$
- Equilibrium Analysis
  - Given player 2's strategy q_2, the best response of player 1 is:

$$q_1^* = B_1(q_2) = \frac{a - q_2 - c_1}{2},$$

  - Given player 1's strategy q_1, the best response of player 2 is:

$$q_2^* = B_2(q_1) = \frac{a - q_1 - c_2}{2},$$

  - A strategy profile (q_1^*, q_2^*) is an Nash equilibrium if every player's strategy is the best response to others' strategies:

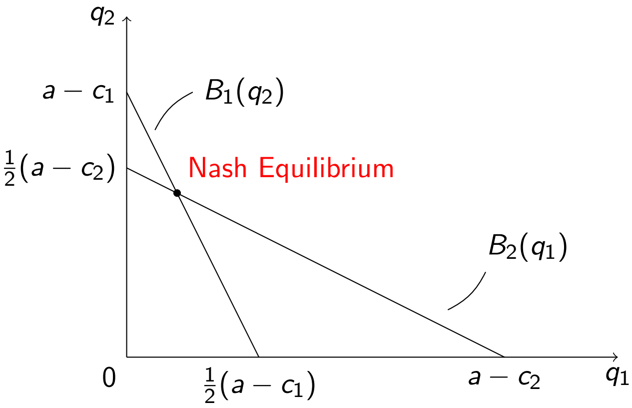
$$q_1^* = B_1(q_2^*), \quad \text{and} \quad q_2^* = B_2(q_1^*)$$

  - Nash Equilibrium:

$$q_1^* = \frac{a + c_1 + c_2}{3} - c_1, \quad q_2^* = \frac{a + c_1 + c_2}{3} - c_2$$

The Cournot Model

- Illustration of Equilibrium
  - Geometrically, the Nash equilibrium is the intersection of both players' best response curves.



The Bertrand Model

- The Bertrand model describes interactions among firms (sellers) who *set prices independently and simultaneously*, under which the customers (buyers) choose quantities accordingly.
- Key features
  - At least two firms producing homogeneous products;
  - Firms do not cooperate, i.e., there is no collusion;
  - Firms compete by setting prices simultaneously;
  - Consumers buy products from a firm with a lower price.
    - If firms charge the same price, consumers randomly select among them.
  - The firms are economically rational and act strategically, seeking to maximize profits given their competitors' decisions.

The Bertrand Model

- Example: The Bertrand Game
  - Two firms decide their respective prices simultaneously;
  - The consumers buy products from a firm with a lower price.

- Game Formulation
  - The set of players is $\mathcal{I} = \{1, 2\}$,
  - The strategy set available to each player $i \in \mathcal{I}$ is the set of all nonnegative real numbers, i.e., $p_i \in [0, \infty)$,
  - The payoff received by each player i is a function of both players' strategies, defined by

$$\Pi_i(p_i, p_{-i}) = (p_i - c_i) \cdot D_i(p_1, p_2)$$

    - ★ c_i is the unit producing cost;
    - ★ $D_i(p_1, p_2)$ is the consumers' demand to player i: (i) $D_i(p_1, p_2) = 0$ if $p_i > p_{-i}$; (ii) $D_i(p_1, p_2) = D(p_i)$ if $p_i < p_{-i}$; (iii) $D_i(p_1, p_2) = D(p_i)/2$ if $p_i = p_{-i}$.

The Bertrand Model

- Assume same cost: $c_1 = c_2 = c$
- Equilibrium Analysis
  - Given player 2's strategy p_2, the best response of player 1 is to select a price p_1 slightly lower than p_2 under the constraint that $p_1 \geq c$:

$$p_1^* = B_1(p_2) = \max\{c,\ p_2 - \epsilon\}$$

  - Given player 1's strategy p_1, the best response of player 2 is to select a price p_2 slightly lower than p_1 under the constraint that $p_2 \geq c$:

$$p_2^* = B_2(p_1) = \max\{c,\ p_1 - \epsilon\}$$

  - Both players will gradually decrease their prices, until reaching the producing cost c. Therefore, the Nash equilibrium is

$$p_1^* = \ p_2^* = c.$$

The Hotelling Model

- The Hotelling model studies *the effect of locations on the price competition* among two or more firms.
- Key features
  - Two firms at different locations sell the homogeneous good;
  - The customers are uniformly distributed between two firms.
  - Customers incur a transportation cost as well as a purchasing cost.
  - The firms are economically rational and act strategically, seeking to maximize profits given their competitors' decisions.

The Hotelling Model

- Example: The Hotelling Game
  - Two firms at different locations decide their respective prices simultaneously;
  - The consumers buy products from a firm with a lower total cost, including both the transportation cost and the purchasing cost.

- Game Formulation
  - The set of players is $\mathcal{I} = \{1, 2\}$, each locating at one end of the interval [0, 1];
  - The strategy set available to each player $i \in \mathcal{I}$ is the set of all nonnegative real numbers, i.e., $p_i \in [0, \infty)$;
  - The payoff received by each player i is a function of both players' strategies, defined by

$$\Pi_i(p_i, p_{-i}) = (p_i - c_i) \cdot D_i(p_1, p_2)$$

    - ★ c_i is the unit producing cost;
    - ★ $D_i(p_1, p_2)$ is the ratio of consumers coming to player i.

The Hotelling Model

- Consumer Demand: $D_i(p_1, p_2)$
  - Under price profile (p_1, p_2), the total cost of a consumer at location $x \in [0, 1]$ buying products from player 1 or 2 is

$$C_1(x) = p_1 + w \cdot x, \quad \text{and} \quad C_2(x) = p_1 + w \cdot (1 - x)$$

  - Under (p_1, p_2), two players receive the following consumer demand:

$$D_1(p_1, p_2) = \frac{p_2 - p_1 + w}{2w}, \quad D_2(p_1, p_2) = \frac{p_1 - p_2 + w}{2w}$$

The Hotelling Model

- Equilibrium Analysis
  - Given player 2's strategy p_2, the best response of player 1 is

$$p_1^* = B_1(p_2) = \frac{p_2 + w + c_1}{2}$$

  - Given player 1's strategy p_1, the best response of player 2 is

$$p_2^* = B_2(p_1) = \frac{p_1 + w + c_2}{2}$$

  - Nash Equilibrium:

$$p_1^* = \frac{3w + c_1 + c_2}{3} + \frac{c_1}{3}, \quad p_2^* = \frac{3w + c_1 + c_2}{3} + \frac{c_2}{3}.$$

The Hotelling Model

- Illustration of Equilibrium
  - Geometrically, the Nash equilibrium is the intersection of both players' best response curves.